

O. M. BEKETOV NATIONAL UNIVERSITY OF URBAN ECONOMY IN KHARKIV

**Educational and Scientific Institute  
energy, information and transport infrastructure**

APPROVED

Director ESIEINI



**WORK PROGRAMME OF THE ACADEMIC DISCIPLINE**

**PROBABILITY THEORY, PROBABILITY PROCESSES AND  
MATHEMATICAL STATISTICS**


type of discipline, code for EP	<i>compulsory, OK 15</i>
semester	<i>3-й</i>
number of ECTS credits	<i>4</i>
form of final control	<i>exam</i>
language of instruction, teaching and assessment	<i>English</i>
department	<i>computer science and information technology</i>

**for higher education applicants:**

level of higher education	<i>12 Information technology</i>
branch of knowledge	<i>122 Computer Science</i>
specialty	<i>Computer Science</i>
educational programme	
form of study	<i>full-time</i>

**2021 – 2022 ACADEMIC YEAR**

Developers:

Surname and initials	Position, email	Scientific degree, academic title	Signature
Maryna BULAIENKO	Associate Professor, Marina.Bulaenko@kname.edu.ua	PhD in Technical Sciences (Candidate of Technical Sciences), Associate Professor	

The work programme was approved **at the proceedings** of the Department computer science and information technology

Minutes dated «30» august 2021 № 2

Head of the Department \_\_\_\_\_  (Maryna NOVOZHYLOVA)

**The work programme of the discipline corresponds to the Educational Programme «Computer Science»**

Guarantor of the Educational Programme \_\_\_\_\_  (Mykola PAN)

### 1. The purpose of the discipline

The purpose of studying of discipline "Probability theory, probabilistic processes and mathematical statistics" is forming the base of students' mathematical and logical thinking, students acquire basic knowledge of the basics of application of probabilistic and statistical apparatus for solving theoretical and applied problems.

### 2. Interdisciplinary connections

The study of this discipline is directly based on the discipline «Higher mathematics».

### 3. Learning outcomes

Program learning outcome	Teaching methods	Forms of evaluation	Learning outcomes of the discipline
LO3. Use knowledge of patterns of random phenomena, their properties and operations on them, models of random processes and modern software environments to solve problems of statistical data processing and construction of predictive models.	Verbal, visual, practical	Oral questioning, testing in Moodle, Tests, practical skills testing; exam (written on tickets)	<p>Know the basic concepts and theorems of the theory of stochastic phenomena and processes.</p> <p>Know the basic laws of distribution of discrete and continuous random variables.</p> <p>Know the basic elements of correlation theory for research systems of random variables.</p> <p>Know the limit theorems of probability theory.</p> <p>Have an idea of the general concepts of probabilistic processes.</p> <p>Apply standard methods of organizing statistical processing and data analysis.</p>
LO7. Understand the principles of modelling organizational and technical systems and operations; use methods of operations research, solving single- and multicritical optimization problems of linear, integer, nonlinear, stochastic programming.			<p>Have an idea about models and algorithms for numerical solution of professional problems.</p> <p>Be able to perform computational experiments, compare the results of experimental data and solutions.</p>

## **4. Program academic discipline**

### **Module 1. Probability theory, probability processes and mathematical statistics**

#### **Content module 1. Random events. Systems of random variables**

The main provisions and definitions are considered. The classical definition of probability is given. The basic principles of combinatorics, the rule of addition, the rule of multiplication are studied. The main types of combinatorial connections.

Operations on events are studied. Axioms of probability and its properties. Geometric probabilities. Basic theorems of probability theory. Reliability models of technical systems. The formula of total probability. Bayes' formula.

Bernoulli's formula is considered. Local Laplace theorem. Poisson's formula. Laplace integral theorem.

Random variables are studied. Forms of problem of the law of distribution of a discrete random variable. Forms of problem of continuous random variable and its properties. Numerical characteristics of random variables.

The laws of distribution of discrete and continuous random variables are studied. Distributions associated with the normal distribution. Random vectors are studied. Numerical characteristics of a random vector.

#### **Content module 2. Probabilistic processes**

The functions of random arguments are studied. The law of distribution of the function of random arguments is considered. The law of large numbers. Markov inequality. Chebyshev's inequality. The law of large numbers in the form of Chebyshev. The law of large numbers in the forms of Hinchin and Bernoulli. The central limit theorem is given. Lyapunov's theorem. Local Laplace limit theorem. Laplace integral limit theorem. The general concepts of probabilistic processes are studied. Markov processes.

#### **Content module 3. Mathematical statistics. The problem of aligning statistical series**

The basic concepts are studied. The simplest methods of processing the results of observations are studied. Statistical distribution of the sample. Frequency histogram. Frequency range. Statistical distribution function. Numerical characteristics of the statistical distribution.

The problem of equalization of statistical series is considered. Construction of a theoretical probability distribution density curve. Estimates of parameters of random variables.

Testing the hypothesis of consistency of theoretical and statistical distribution. The degree of discrepancy between the theoretical and statistical distributions is studied. The criterion of agreement  $\chi^2$  - Pearson.

## 5. The structure of the discipline and distribution of time

Content modules	Number of hours				
	total	including			
		lectures	practice	lab	independent work
<b>MODULE 1 (3rd semester)</b>	<b>120</b>	<b>30</b>	<b>15</b>	<b>-</b>	<b>75</b>
<b>Content module 1</b>	<b>30</b>	<b>14</b>	<b>7</b>	<b>-</b>	<b>9</b>
<b>Content module 2</b>	<b>30</b>	<b>8</b>	<b>4</b>	<b>-</b>	<b>18</b>
<b>Content module 3</b>	<b>30</b>	<b>8</b>	<b>4</b>	<b>-</b>	<b>18</b>
<b>Individual task</b>	<b>15</b>	-	-	-	<b>15</b>
<b>Final control</b>	<b>15</b>	-	-	-	<b>15</b>

## 6. Topics of lectures

Topic	Contents (plan)	Number of aud. hours
<b>Module 1</b>		
<b>Content module 1</b>		
Empirical and logical foundations of probability theory	Introduction. Basic definitions. Classical definition of probability. Basic principles of combinatorics, rule of addition, rule of multiplication. The main types of combinatorial connections. Conclusions and answers to questions.	<b>2</b>
Basic theorems of probability theory, their technical interpretation	Introduction. Operations on events. Frequency and statistical probability of a random event. Probability axioms. Geometric probabilities. Basic theorems of probability theory. Reliability models of technical systems. The formula of total probability. Bayesian formula. Conclusions and answers to questions.	<b>4</b>
Scheme of independent tests	Introduction. Repeat the experiment. Bernoulli's formula. Local Laplace theorem. Poisson's formula. Laplace integral theorem. The most probable number of events. Conclusions and answers to questions.	<b>2</b>

Distribution laws and numerical characteristics of random variables	<p>Introduction.</p> <p>Random variables, discrete random variable. The law of distribution of a random variable. Forms of problem of the law of distribution of a discrete random variable. Continuous random variable. Forms of problem of continuous random variable and its properties. Numerical characteristics of random variables. Characteristics of the position of a random variable on the numerical axis. Moments of random variables. Properties of moments of random variables.</p> <p>Conclusions and answers to questions.</p>	<b>2</b>
The most common discrete and continuous laws of distribution of random variables	<p>Introduction.</p> <p>Distribution laws of discrete random variables: Bernoulli distribution, binomial distribution law, geometric distribution, hypergeometric distribution, Poisson distribution law. Laws of distribution of continuous random variables: uniform law of distribution, indicative (exponential) law of distribution, normal law of distribution. Distributions associated with the normal distribution: Pearson distribution, Student's distribution, Fisher-Snedekor distribution.</p> <p>Conclusions and answers to questions.</p>	<b>2</b>
Multidimensional random variables	<p>Introduction.</p> <p>Random vectors. Properties of the random vector distribution function. The distribution density of a random vector. Conditional distribution laws. Numerical characteristics of a random vector.</p> <p>Conclusions and answers to questions.</p>	<b>2</b>
<b>Content module 2</b>		
Random argument functions	<p>Introduction.</p> <p>Functions of random arguments. Numerical characteristics of the function of random arguments. Theorems on numerical characteristics of the function of random arguments. The law of distribution of the function of random arguments.</p> <p>Conclusions and answers to questions.</p>	<b>2</b>
Limited theorems of probability theory.	<p>Introduction.</p> <p>The law of large numbers. Markov inequality. Chebyshev's inequality. The law of large numbers in the form of Chebyshev. The law of large numbers in the forms of Hinchin and Bernoulli. Central limit theorem. Lyapunov's theorem. Local Laplace limit theorem. Laplace integral limit theorem.</p> <p>Conclusions and answers to questions.</p>	<b>2</b>

General concepts of probabilistic processes	Introduction. Basic definitions. Laws of distribution of probabilistic processes. Characteristics of probabilistic processes Conclusions and answers to questions.	<b>2</b>
Markov processes	Introduction. Markov probabilistic process with discrete states. Markov probabilistic process with discrete time. Markov probabilistic process with discrete states and continuous time. Conclusions and answers to questions.	<b>2</b>
<b>Content module 3</b>		
Initial evaluation of statistic	Introduction. Basic concepts. The simplest methods of processing the results of observations. Statistical distribution of the sample. Frequency histogram. Frequency range. Statistical distribution function. Numerical characteristics of the statistical distribution. Conclusions and answers to questions.	<b>2</b>
Statistical estimates of distribution parameters	Introduction. Alignment of statistical series. Construction of a theoretical probability distribution density curve. Estimation of parameters of random variables. Evaluation criteria. Estimates for mathematical expectation and variance. Method of moments for point estimation of distribution parameters. The most plausible method. Confidence interval. Confidence probability. Conclusions and answers to questions.	<b>4</b>
Testing statistical hypotheses	Introduction. Testing the hypothesis of consistency of theoretical and statistical distribution. The degree of discrepancy between the theoretical and statistical distributions. Consent criterion. Pearson's criterion $\chi^2$ . Conclusions and answers to questions.	<b>2</b>

### 7. Topics of practical classes

Topic	Contents (plan)	Number of aud. hours
<b>Module 1</b>		
<b>Content module 1</b>		
Elements of combinatorics, direct probability	Introduction. Theoretical part. Elements of combinatorics. Direct probability calculation.	<b>1</b>

calculation.	Tasks for practical training. Control questions.	
Basic theorems. Reliability models of technical systems.	Introduction. Theoretical part. Basic theorems (addition, multiplication). Reliability models of technical systems. Tasks for practical training. Control questions.	<b>1</b>
Full probability formula, hypothesis theorem.	Introduction. Theoretical part. The formula of total probability. Bayesian formula. Tasks for practical training. Control questions.	<b>1</b>
Repeat the experiment.	Introduction. Theoretical part. Bernoulli's formula. Local and integral Laplace theorems. The most probable number of events. Tasks for practical training. Control questions.	<b>1</b>
Discrete random variable.	Introduction. Theoretical part. Discrete random variable. Tasks for practical training. Control questions.	<b>1</b>
Continuous random variable.	Introduction. Theoretical part. Continuous random variable. Tasks for practical training. Control questions.	<b>1</b>
Separate discrete and continuous laws of distribution of random variables.	Introduction. Theoretical part. Distribution laws of discrete random variables: Bernoulli distribution, binomial distribution law, geometric distribution, hypergeometric distribution, Poisson distribution law. Laws of distribution of continuous random variables: uniform law of distribution, indicative (exponential) law of distribution, normal law of distribution. Tasks for practical training. Control questions.	<b>1</b>
<b>Content module 2</b>		
Function of random variables	Introduction. Theoretical part. The functions of random arguments. Numerical characteristics of the function of random arguments. Tasks for practical training. Control questions.	<b>2</b>
General concepts of probabilistic processes.	Introduction. Theoretical part. Laws of distribution of probabilistic processes. Characteristics of probabilistic processes. Tasks for practical training. Control questions.	<b>2</b>



<b>Content module 3</b>		
The simplest methods of processing the results of observations.	Introduction. Theoretical part. Statistical distribution of the sample. Frequency histogram. Frequency range. Statistical distribution function. Numerical characteristics of the statistical distribution. Tasks for practical training. Control questions.	<b>1</b>
Alignment of statistical series.	Introduction. Theoretical part. Construction of a theoretical probability distribution density curve. Tasks for practical training. Control questions.	<b>1</b>
Estimation of parameters of random variables.	Introduction. Theoretical part. Estimation of parameters of random variables. Confidence interval. Tasks for practical training. Control questions.	<b>1</b>
Estimation of parameters of random variables. Test of statistical hypotheses.	Introduction. Theoretical part. Testing the hypothesis of consistency of theoretical and statistical distribution. Pearson's criterion $\chi^2$ . Tasks for practical training. Control questions.	<b>1</b>

### **8. Individual task (IT)**

Type: calculation and graphic work.

Title: Mathematical processing of statistical data.

Purpose: students acquire knowledge and skills in the field of statistical data processing.

### **9. Methods of control and the procedure for assessing learning outcomes**

The current control system is based on the use of the following forms of control:

- oral examination based on lecture materials;
- oral examination based on the results of the practical lesson;
- performance of control works;
- testing in a virtual educational environment on the MOODLE platform version 3.9.
- protection of the report on calculation graphic work.

The final control in the form of an examination is carried out in writing on examination tickets.

### **Structure of the discipline and the distribution of points**

Content modules	Maximum number of points
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	total	practice	lab.	independent work
<b>MODULE 1 (3rd semester)</b>	<b>100</b>	<b>45</b>	-	<b>30</b>
<b>Content module 1</b>	<b>20</b>	<b>15</b>	-	<b>5</b>
<b>Content module 2</b>	<b>20</b>	<b>15</b>	-	<b>5</b>
<b>Content module 3</b>	<b>15</b>	<b>10</b>	-	<b>5</b>
<b>Individual task</b>	<b>15</b>	-	-	<b>15</b>
<b>Final control</b>	<b>30</b>	-	-	-

### Types of tasks, means of control and the maximum number of points

Types of tasks and means of control (testing, tests, individual tasks, reports on laboratory classes)	Distribution of points
<b>Module 1</b>	
<b>Content module 1</b>	<b>20</b>
Practical exercise "Direct probability calculation" (performed calculation on the task, oral interview).	1
Test work №1.	2
Tasks for independent work 1.	2
Practical exercise "Basic theorems" (calculation of the problem, oral examination).	1
Test work №2.	3
Practical exercise "Models of reliability of technical systems" (performed calculation on the task, oral examination).	1
Test work №3.	5
Practical exercise "Full probability formula, hypothesis theorem. Repetition of the experiment" (performed calculation on the task, oral examination).	1
Tasks for independent work 2.	3
Practical exercise "Some discrete and continuous laws of distribution of random variables" (performed calculation on the task, oral examination).	1
<b>Content module 2</b>	<b>20</b>
Test work №4.	4
Practical exercise "Numerical characteristics of the function of random arguments" (performed calculation on the task, oral examination).	1
Tasks for independent work 3.	2
Test work №5.	5
Tasks for independent work 4.	3
Test work №6.	4
Practical exercise "Characteristics of probabilistic processes" (calculation of the task, oral interview).	1
<b>Content module 3</b>	<b>15</b>
Practical exercise "The simplest methods of processing the results of	5

observations" (performed calculation on the task).	
Practical exercise "Estimation of parameters of random variables" (performed calculation on the task).	5
Tasks for independent work 5.	5
<b>Individual task (CGW)</b>	<b>15</b>
Preparation of the calculation part	10
Presentation and defense	5
<b>Final control - exam</b>	<b>30</b>
Theoretical question 1	6
Theoretical question 2	6
Task 1	6
Task 2	6
Task 3	6
<b>TOTAL FOR THE MODULE</b>	<b>100</b>

### Grading scale

The sum of points for all types of educational activities	Score on a national scale	
	for the exam, diff. offset	for test
90-100	excellent	passed
82-89	good	
74-81		
64-73	satisfactorily	
60-63		
35-59	unsatisfactory with the possibility of retaking	failed with the possibility of retaking
0-34	unsatisfactory with mandatory re-study of the discipline	failed with mandatory re-study of the discipline

## 10. Material and technical and information support

### Methodical support

1. Булаєнко М. В. Теорія ймовірностей та математична статистика : навч. посібник: [у 2 ч.] Частина 1 / М. В. Булаєнко, О. Б. Костенко ; Харків. нац. ун-т міськ. гос-ва ім. О. М. Бекетова. – Харків : ХНУМГ ім. О. М. Бекетова, 2019. – ч.1, 172 с.

[https://teams.microsoft.com/l/file/BDD4229B-4912-4FDE-A01A-CADCB4DE7E6E?tenantId=b4e18cf3-2cc0-446f-afb7-f3c65cf9d6d8&fileType=pdf&objectUrl=https%3A%2F%2Fknameedu.sharepoint.com%2Fsites%2F122126--%2FShared%20Documents%2FGeneral%2F%D0%9C%D0%B5%D1%82%D0%BE%D0%B4%D0%B8%D1%87%D0%BD%D1%96%20%D0%BC%D0%B0%D1%82%D0%B5%D1%80%D1%96%D0%B0%D0%B%D0%B8%2F%D0%9D%D0%9F%20%D0%A2%D0%99%2C%D0%99%D0%9F%D1%82%D0%B0%D0%9C%D0%A1\(%D1%871\)\\_126.pdf&baseurl=https%3A%2F%2Fknameedu.sharepoint.com%2Fsites%2F122126--](https://teams.microsoft.com/l/file/BDD4229B-4912-4FDE-A01A-CADCB4DE7E6E?tenantId=b4e18cf3-2cc0-446f-afb7-f3c65cf9d6d8&fileType=pdf&objectUrl=https%3A%2F%2Fknameedu.sharepoint.com%2Fsites%2F122126--%2FShared%20Documents%2FGeneral%2F%D0%9C%D0%B5%D1%82%D0%BE%D0%B4%D0%B8%D1%87%D0%BD%D1%96%20%D0%BC%D0%B0%D1%82%D0%B5%D1%80%D1%96%D0%B0%D0%B%D0%B8%2F%D0%9D%D0%9F%20%D0%A2%D0%99%2C%D0%99%D0%9F%D1%82%D0%B0%D0%9C%D0%A1(%D1%871)_126.pdf&baseurl=https%3A%2F%2Fknameedu.sharepoint.com%2Fsites%2F122126--)

<https://eprints.kname.edu.ua/57525/1/2016%20%D0%BF%D0%B5%D1%87.%20404%D0%9C%20%D0%9C%D0%A0%20%D0%B4%D0%BE%20%D0%BF%D1%80%2C%20%D1%81%D0%B0%D0%BC.pdf>

2. Булаєнко М.В. Методичні рекомендації до проведення практичних занять та виконання самостійної роботи з дисципліни «Теорія ймовірностей, ймовірнісні процеси та математична статистика» (для студентів 2 курсу денної форми навчання освітньо-кваліфікаційного рівня «бакалавр», спеціальності 126 – Інформаційні системи та технології) / Харків. нац. ун-т міськ. госп-ва ім. О. М. Бекетова ; уклад. М. В. Булаєнко – Харків : ХНУМГ ім. О. М. Бекетова, 2020. – 59 с.

<https://dl.kname.edu.ua/course/view.php?id=2221> Moodle virtual learning environment, version 3.9.

3. Bulaienko M.V. Distance course "Probability Theory, Probabilistic Processes and Mathematical Statistics". -[Electronic resource] - Access mode <https://dl.kname.edu.ua/course/view.php?id=2221> Moodle virtual learning environment, version 3.9.

### Recommended literature and information resources

1. Виктор Барковский, Нина Барковская, Алексей Лопатин. Теорія ймовірностей та математична статистика Навчальний посібник / В. Барковский - К: Видавництво Центр навчальної літератури, 2019.-424 с.

2. Огірко О. І., Галайко Н. В. Теорія ймовірностей та математична статистика: навчальний посібник / О. І. Огірко, Н. В. Галайко. – Львів: ЛьвДУВС, 2017. – 292 с.

3. GeoGebra Динамічна математика для навчання та викладання [Електронний ресурс] : [Веб-сайт]. – Режим доступу : <http://www.geogebra.org/>

4. Табличний процесор Google-таблиці. – Режим доступу:

<https://docs.google.com/spreadsheets/u/0/>.

### Hardware, hardware, software products

Name of computer laboratory	Model and brand of personal computers, their number	Name of application packages (including licensed)	Internet access, availability of access channels (yes / no)
Laboratory of Informatics and Computer Engineering	Impression P + computer - 18 units. multimedia projector	- ESET Antivirus Software - Office Pro 2013 Rus OLP NL Academy.	so